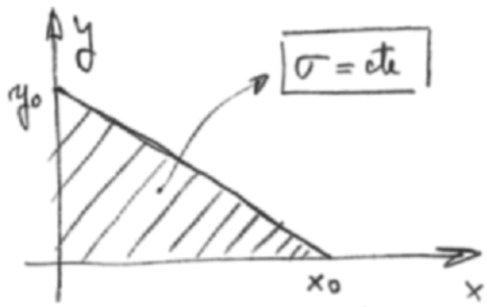
CALCOLARE IL CENTRO DI MASSA PER I SEGUENTI DISEGNI :

a)



; σ : densità di massa per unità di Area. ; $[\sigma] = \frac{kg}{m^2}$

SOLUZIONE

LA LINEA CHE VA DA y_0 a x_0 HA UNA EQ.: $\boxed{y = -\frac{y_0}{x_0}x + y_0.}$

$\boxed{dA = y\,dx}$

$$x_{cm} = \frac{\int x\, dA\, \cancel{\sigma}}{\int dA\, \cancel{\sigma}} \quad (\sigma \to cte) \quad = \frac{\int_0^{x_0}\left(-\frac{y_0}{x_0}x + y_0\right)x\,dx}{\int_0^{x_0}\left(-\frac{y_0}{x_0}x + y_0\right)dx} =$$

$$= \frac{-\frac{y_0}{x_0}\frac{x_0^3}{3} + y_0\frac{x_0^2}{2}}{-\frac{y_0}{x_0}\frac{x_0^2}{2} + y_0 x_0} = \frac{x_0}{3}$$

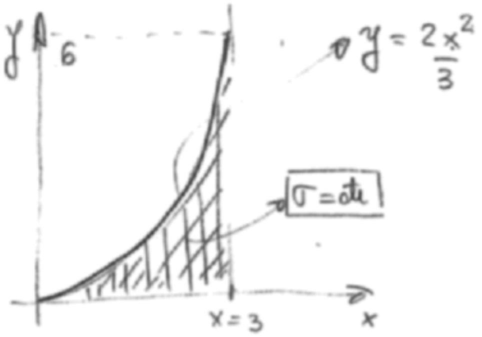
$$y_{cm} = \frac{\int y\, dA\, \sigma}{\int dA\, \sigma} = \frac{\int_0^{y_0}\left(y x_0 - \frac{x_0}{y_0}y^2\right)dy}{\int_0^{y_0}\left(x_0 - \frac{x_0}{y_0}y\right)dy} = \frac{\frac{y_0^2 x_0}{2} - \frac{x_0 y_0^3}{3 y_0}}{x_0 y_0 - \frac{x_0 y_0^2}{2 y_0}} = \frac{y_0}{3}$$

Cambio variabile $\to dA = x\,dy$

$\boxed{x = x_0 - \frac{x_0}{y_0}y}$

$\boxed{\bar{r}_{cm} = \left(\frac{x_0}{3}, \frac{y_0}{3}\right)}$

b)



$dA = y\,dx$

$$x_{cm} = \frac{\int_0^3 x\,dA\,\cancel{\sigma}}{\int_0^3 dA\,\cancel{\sigma}} = \frac{\int_0^3 \frac{2}{3}x^3 dx}{\int_0^3 \frac{2}{3}x^2 dx} = \frac{\cancel{\frac{2}{3}}\frac{81}{4}}{\cancel{\frac{2}{3}}\frac{27}{3}} = \frac{9}{4}$$

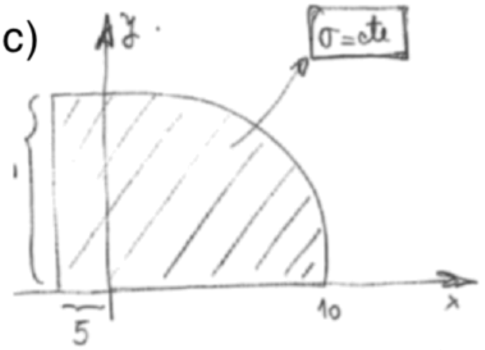
$$y_{cm} = \frac{\int_0^6 y\,dA\,\cancel{\sigma}}{\int_0^6 dA\,\cancel{\sigma}} = \frac{\int_0^3 \frac{8}{9}x^3(3-x)\,dx}{\frac{2}{3}\frac{27}{3}} = \frac{54/5}{6} = \frac{9}{5}$$

$\boxed{dA = (3-x)\,dy = (3-x)\frac{4}{3}x\,dx}$

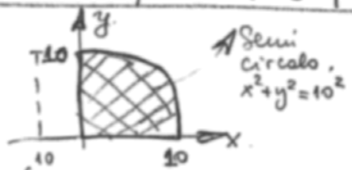
cambio variabile

$\boxed{\vec{r}_{cm} = \left(\frac{9}{4}, \frac{9}{5}\right)}$

c)



Prima facciamo:

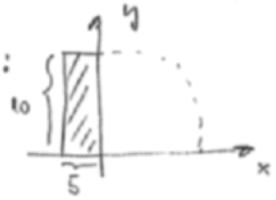


$$x_{cm}^{semi} = \frac{\int_0^{10} x\,dA\,\cancel{\sigma}}{\int_0^{10} dA\,\cancel{\sigma}} = \frac{\int_0^{10} x\sqrt{10^2-x^2}\,dx}{\pi\,10^2/4} = \frac{10^3/3}{\pi\,10^2/4} = \frac{40}{3\pi}$$

$$dA = y\,dx = \sqrt{10^2-x^2}\,dx$$

$\boxed{\text{Non è necessario fare il conto. Solo per simmetria}}$ → $\boxed{y_{cm}^{semi} = \frac{40}{3\pi}}$

Ora facciamo:



$\boxed{\begin{array}{l} x_{cm} = -2.5 \\ y_{cm} = 5 \end{array}}$ → per simmetria. ($\sigma = cte$)

allora facciamo:

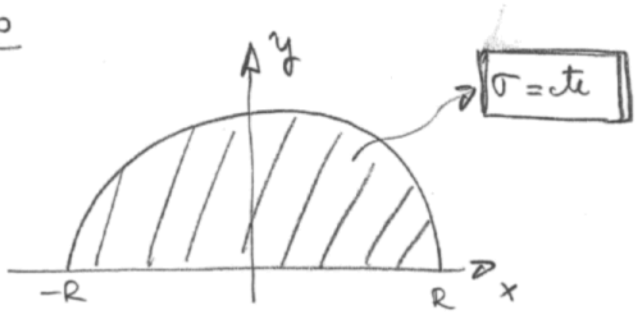
$$\vec{r}_{cm} = \frac{\text{Massa}^{semi}\,\vec{r}_{cm}^{\,semi} + \text{Massa}^{▯}\,\vec{r}_{cm}^{\,▯}}{M_{TOTALE}}$$

$$\Rightarrow \quad y_{cm} = \frac{(25\pi)\sigma\left(\frac{40}{3\pi}\right) + (50)\sigma(5)}{(25\pi+50)\sigma} \sim 4.54$$

$\Rightarrow \boxed{\vec{r}_{cm} \simeq (1{,}62;\ 4.54)}$

$$x_{cm} = \frac{(\cancel{25}\pi)\cancel{\sigma}\left(\frac{40}{3\pi}\right) + (\cancel{50})\cancel{\sigma}(-2.5)}{(\cancel{25}\pi + \cancel{50})\cancel{\sigma}} = \frac{\frac{40}{3} - 5}{(2+\pi)} \sim 1.62$$

d) Semicircolo



$$\boxed{M = \int_0^R \int_0^{\pi} \sigma \, dr \, r \, d\theta = \frac{\pi R^2}{2}\sigma}$$

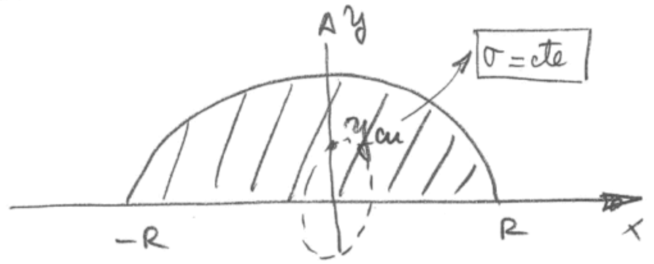
$$x_{cm} = \int_0^{\sqrt{R^2-x^2}} \sigma \int_{-R}^{R} x \, \frac{dx\,dy}{M} = \frac{\sigma}{M}\int_{-R}^{R} \underbrace{x}_{\text{dispari}} \overbrace{\sqrt{R^2-x^2}}^{\text{pari}} \, dx = 0$$

che per simmetria era evidente.

$$y_{cm} = \frac{\sigma}{M}\int_0^{\sqrt{R^2-x^2}} \int_{-R}^{R} y \, dx \, dy = \frac{2\sigma}{M}\int_0^R dy \int_0^{\sqrt{R^2-y^2}} y \, dx = \frac{4}{3}\frac{R}{\pi}$$

$$\boxed{\vec{r}_{cm} = \left(0, \frac{4}{3}\frac{R}{\pi}\right)}$$

e) Semicircolo con TEOREMA DI PAPPUS - GULDINO



$\boxed{\sigma = cte}$

asse di Rotazione

$\boxed{\text{Per simmetria } x_{cm} = 0}$

$\boxed{y_{cm} = ?}$ $\longrightarrow$ Volume = $\text{Area} \times$ (percorso del y_{cm})

fatto con la Rotazione.

dell'oggetto

$$\left(\frac{4}{3}\pi R^3\right) = \frac{\pi R^2}{2} \times \underbrace{(2\pi\, y_{cm})}$$

con la Rotazione y_{cm} fa un circolo.

$$\frac{4}{3} R = \pi\, y_{cm}$$

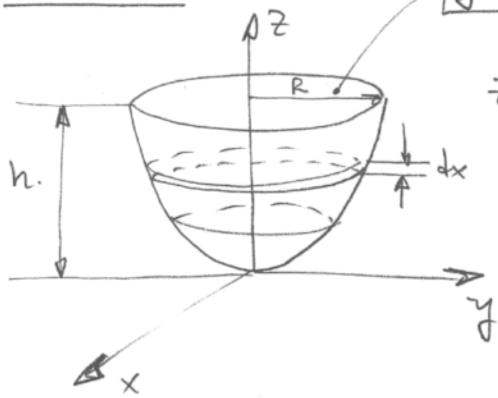
$$\Rightarrow \boxed{y_{cm} = \frac{4R}{3\pi}.}$$

$$\boxed{\vec{r}_{cm} = \left(0, \frac{4R}{3\pi}\right)}$$

f)

PARABOLOIDE

$\boxed{\rho = \text{cost}}$



$z = x^2 + y^2.$

$\boxed{\rho = \text{densità di massa per unità di volume} ; [\rho] = \frac{kg}{m^3}}$

$h = R^2$

$$\boxed{M = \int \rho \, dv = \int_0^h \rho \pi z \, dz = \rho \pi \frac{h^2}{2} = \rho \frac{h R^2 \pi}{2}} \checkmark$$

$$\boxed{dv = z \, dz \, \pi}$$

$$\boxed{\text{Per simmetria} \quad x_{cm} = y_{cm} = 0}$$

$$\boxed{V_{parab} = \int_0^{2\pi} \int_0^R r^2 \, r \, dr \, d\theta = \frac{R^4}{4} 2\pi = \frac{\pi h R^2}{2}} \checkmark$$

$z = r^2$

$dz = 2r\,dr.$

$\pi r^2 dz = 2\pi r^3 dr.$

$$\int_0^R r^3 dr = 2\pi \frac{R^4}{4} = \frac{\pi R^2 h}{2}. \checkmark$$

guardate che il volume del Paraboloide è lo stesso che il Volume fra il Paraboloide e il piano $z=0$.

$V_{sotto Parab} = \frac{\pi h R^2}{2}$

$V_{cilindro} = \pi R^2 h$

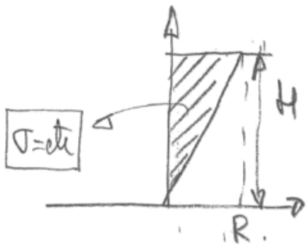
$\Rightarrow V_{parab} = \pi R^2 h - \frac{\pi h R^2}{2}$

$$\boxed{V_{parab} = \frac{\pi R^2 h}{2}}$$

$$z_{cm} = \frac{\int z \, dv \rho}{M} = \frac{\int_0^h z^2 \rho \pi \, dz}{\frac{\rho h R^2 \pi}{2}} = \frac{h^3 \pi / 3}{\frac{h R^2 \pi}{2}} = \frac{2h}{3}.$$

$$\boxed{\bar{r}_{cm} = \left(0, 0, \frac{2h}{3}\right)}$$

TRIANGOLO (NORMALE E CON GOLDINO)



$y = \frac{H}{R}x \Rightarrow \boxed{M = \frac{HR}{2}\sigma}$

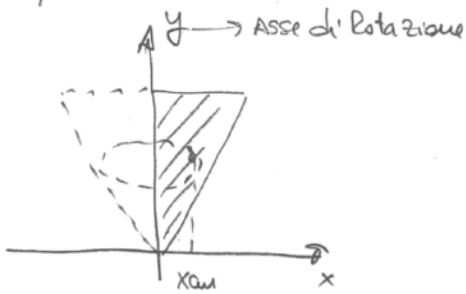
$$\boxed{x_{cm} = \int_0^R \frac{(H-y)\,dx\,\sigma}{M} = \frac{R}{3}}$$

$$\boxed{dA = (H-y)\,dx}$$

$$\boxed{y_{cm} = \frac{\int_0^H \frac{R}{H} y^2\,dy\,\sigma}{M} = \frac{2}{3}H}$$

$$\vec{r}_{cm} = \left(\frac{R}{3}, \frac{2}{3}H\right)$$

La xcm anche si può fare con GULDINO.



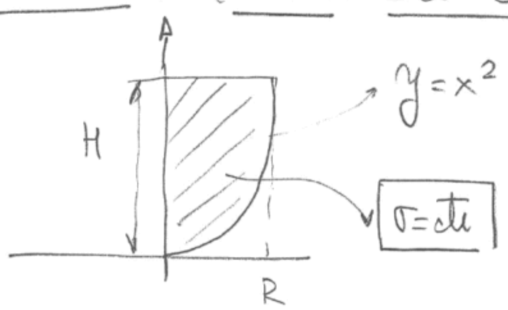
quando il Triangolo Rota fa un CONO ⇒ Volume del CONO = Area del Triangolo × (percorso x cm)

$$\frac{R^{\cancel{2}}\cancel{H}\cancel{\pi}}{3} = \frac{\cancel{R}\cancel{H}}{\cancel{2}}\left(\cancel{2\pi}\,x_{cm}\right)$$

$$\boxed{x_{cm} = \frac{R}{3}}$$

Compiti per CASA: FARE LO STESSO PER y cm h)

METTA DI PARABOLA (NORMALE E CON GOLDINO)



$$\boxed{H = R^2}$$

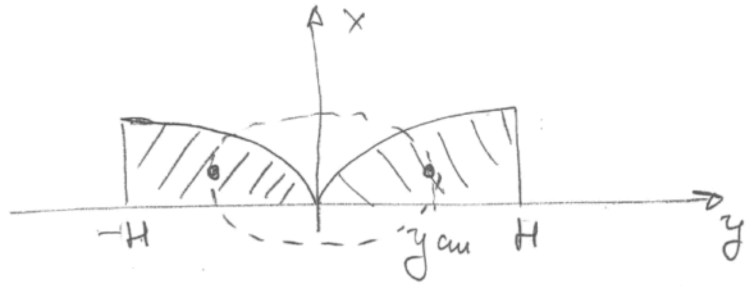
$$\boxed{M = \sigma \int_0^R (H - x^2)\, dx = 2\sigma \frac{R^3}{3}}$$

$$\boxed{X_{cm} = \frac{\int_0^R \sigma (Hx - x^3)\, dx}{M} = \sigma \frac{\frac{HR^2}{2} - \frac{R^4}{4}}{\frac{2R^3}{3}\cancel{\sigma}} = \frac{3R}{8}}$$

$$\boxed{M = \sigma \int_0^H y^{1/2}\, dx = \frac{2\sigma}{3} H^{3/2} = \frac{2\sigma}{3} R^3 \checkmark}$$

$$\boxed{y_{cm} = \frac{\int_0^H y^{3/2}\, dy\, \sigma}{M} = \frac{\frac{\cancel{2}}{5} R^5 \cancel{\sigma}}{\frac{\cancel{2}R^3}{3}\cancel{\sigma}} = \frac{3}{5} R^2 = \frac{3}{5} H}$$

La y_{cm} viene anche calcolata con GULDINO.



$$\text{Volume} = 2\pi \int_0^H dr\, r\, (r^{1/2}) = 2\pi \int_0^H r^{3/2}\, dr = 2\pi \frac{2}{5} H^{5/2} = \frac{4\pi}{5} H^{5/2} = \frac{4\pi R H^2}{5}$$

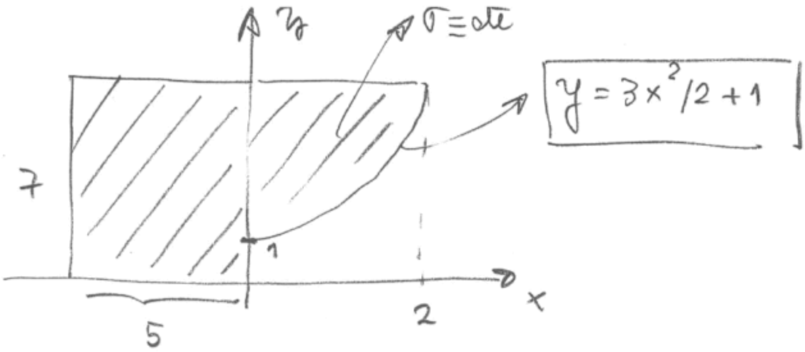
$$\text{Area} = \frac{2\sigma}{3} R^3$$

$$\text{Volume} = \text{Area} \times (2\pi y_{cm})$$

$$\frac{\cancel{4\pi} R H^2}{5} = \frac{\cancel{2}}{3} R^3 \times (\cancel{2\pi}\, y_{cm})$$

$$\boxed{\frac{3}{5} H = y_{cm}} \checkmark$$

i)



$$\left\{\begin{array}{l} x_r^{cm} = -2.5 \\ y_r^{cm} = 3.5 \end{array}\right. \quad ; \quad \left.\begin{array}{l} x_{cm}^{Parabola} = \frac{3}{4} \\ y_{cm}^{Parabola} = \frac{23}{5} \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} x_{cm} = -\frac{163}{86} \\ y_{cm} = \frac{1593}{430} \end{array}}$$